

7.3

SOLVE

$$\begin{aligned} 4x + 6y - 3z &= -5 \\ 3x + 4y + z &= 11 \\ -x - 2y + z &= 1 \end{aligned}$$

$$\begin{array}{cccc} x & y & z & = & c \\ \left[ \begin{array}{ccc|c} 4 & 6 & -3 & -5 \\ 3 & 4 & 1 & 11 \\ \textcircled{-1} & -2 & 1 & 1 \end{array} \right] & R_3 \leftrightarrow R_1 & \text{PIVOT} \end{array}$$

SWAP

$$\begin{array}{cccc} \left[ \begin{array}{ccc|c} \textcircled{-1} & -2 & 1 & 1 \\ 3 & 4 & 1 & 11 \\ 4 & 6 & -3 & -5 \end{array} \right] & \begin{array}{l} \text{PIVOT} \\ (-1)R_1 \end{array} \end{array}$$

SCALE

$$\begin{array}{cccc} \left[ \begin{array}{ccc|c} \textcircled{1} & 2 & -1 & -1 \\ 3 & 4 & 1 & 11 \\ 4 & 6 & -3 & -5 \end{array} \right] & \begin{array}{l} \text{NEGATIVE OF ENTRY TO BE ELIMINATED} \\ R_2 + (-3)R_1 \\ R_3 + (-4)R_1 \end{array} \end{array}$$

REPLACE

$$\begin{array}{cccc} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & \textcircled{-2} & 4 & 14 \\ 0 & -2 & 1 & -1 \end{array} \right] & (-\frac{1}{2})R_2 \end{array}$$

$$\left[ \begin{array}{ccc|c} \textcircled{1} & 2 & -1 & -1 \\ 0 & \textcircled{1} & -2 & -7 \\ 0 & 0 & \textcircled{1} & 5 \end{array} \right]$$

REF

$$\begin{aligned} x + 2y - z &= -1 \\ y - 2z &= -7 \\ z &= 5 \\ y - 2(5) &= -7 \rightarrow y = 3 \\ x + 2(3) - 5 &= -1 \rightarrow x = -2 \\ \boxed{(x, y, z) = (-2, 3, 5)} \end{aligned}$$

$$\begin{array}{cccc} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & \textcircled{1} & -2 & -7 \\ 0 & -2 & 1 & -1 \end{array} \right] & R_3 + (2)R_2 \end{array}$$

$$\begin{array}{cccc} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & \textcircled{-3} & -15 \end{array} \right] & (-\frac{1}{3})R_3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 5 \end{bmatrix} \begin{array}{l} R_1 + (1)R_3 \\ R_2 + (2)R_3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix} R_1 + (-2)R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \rightarrow \begin{array}{l} x = -2 \\ y = 3 \\ z = 5 \end{array}$$

RREF  $(x, y, z) = (-2, 3, 5)$

CHECK:

$$\begin{array}{l} 4(-2) + 6(3) - 3(5) = -8 + 18 - 15 = -5 \checkmark \\ 3(-2) + 4(3) + (5) = -6 + 12 + 5 = 11 \checkmark \\ -(-2) - 2(3) + (5) = 2 - 6 + 5 = 1 \checkmark \end{array}$$

↑  
REQUIRED

$$\begin{array}{c} x \quad y \quad z = c \\ \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow$$

RREF

$$\begin{array}{l} x + 3z = 5 \\ y - 2z = 1 \end{array} \rightarrow \begin{array}{l} x = -3z + 5 \\ y = 2z + 1 \\ z = z \end{array}$$

$$\left. \begin{array}{l} x = -3t + 5 \\ y = 2t + 1 \\ z = t \end{array} \right\} \text{PARAMETRIC SOLUTION}$$

$$t \in (-\infty, \infty)$$

INFINITELY MANY SOLUTIONS

## 8.1 MATRICES

RECTANGULAR ARRAY OF NUMBERS

$$\begin{array}{l} \text{ROW 1} \rightarrow \\ \text{ROW 2} \rightarrow \\ \text{ROW 3} \rightarrow \end{array} \left[ \begin{array}{cccc} 3 & 4 & -11 & -17 \\ 2 & 1 & -4 & 5 \\ -1 & -2 & 5 & -9 \end{array} \right] = A$$

↑  
COLUMN 1

↑  
COLUMN 2

↑  
COLUMN 3

↑  
COLUMN 4

$a_{ij}$  = ENTRY IN ROW  $i$ , COLUMN  $j$

$a_{2,3}$  = ENTRY IN ROW 2, COLUMN 3 = -4

$$a_{1,4} = -17$$